Tiled Image Convention for Storing Compressed Images in FITS Binary Tables

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1 General Description

This document describes a convention for compressing n-dimensional images and storing the resulting byte stream in a variable-length column in a FITS binary table. The FITS file structure outlined here is independent of the specific data compression algorithm that is used. The implementation details for 4 widely used compression algorithms are described here, but any other compression technique could also be supported by this convention.

The general principle used in this convention is to first divide the n-dimensional image into a rectangular grid of subimages or 'tiles'. Each tile is then compressed as a block of data, and the resulting compressed byte stream is stored in a row of a variable length column in a FITS binary table. By dividing the image into tiles it is generally possible to extract and uncompress subsections of the image without having to uncompress the whole image. The default tiling pattern treats each row of a 2-dimensional image (or higher dimensional cube) as a tile, such that each tile contains `NAXIS1` pixels. This default many not be optimal for some applications or compression algorithms, so any other rectangular tiling pattern may be defined using the `ZTILEn` keywords that are described below. In the case of relatively small images it may be sufficient to compress the entire image as a single tile, resulting in an output binary table with 1 row. In the case of 3-dimensional data cubes, it may be advantageous to treat each plane of the cube as a separate tile if application software typically needs to access the cube on a plane by plane basis.

2 Keywords

The following keywords are defined by this convention for use in the header of the FITS binary table extension to describe the structure of the compressed image.

- `ZIMAGE` (required keyword) This keyword must have the logical value T. It indicates that the FITS binary table extension contains a compressed image, and that logically this extension should be interpreted as an image and not as a table.
- `ZCMPTYPE` (required keyword) The value field of this keyword shall contain a character string giving the name of the algorithm that must be used to decompress the image. Currently, values of `GZIP_1`, `RICE_1`, `PLIO_1`, and `HCOMPRESS_1` are reserved, and the corresponding algorithms are described in a later section of this document.
- `ZBITPIX` (required keyword) The value field of this keyword shall contain an integer that gives the value of the `BITPIX` keyword in the uncompressed FITS image.

- ZNAXIS (required keyword) The value field of this keyword shall contain an integer that gives the value of the NAXIS keyword in the uncompressed FITS image.

- ZNAXISn (required keywords) The value field of these keywords shall contain a positive integer that gives the value of the NAXISn keywords in the uncompressed FITS image.

- ZTILEn (optional keywords) The value of these indexed keywords (where n ranges from 1 to ZNAXIS) shall contain a positive integer representing the number of pixels along axis n of the compression tiles. Each tile of pixels is compressed separately and stored in a row of a variable-length vector column in the binary table. The size of each image dimension (given by ZNAXISn) is not required to be an integer multiple of ZTILEn, and if it is not, then the last tile along that dimension of the image will contain fewer image pixels than the other tiles. If the ZTILEn keywords are not present then the default 'row by row' tiling will be assumed such that ZTILE1 = ZNAXIS1, and the value of all the other ZTILEn keywords equals 1.

  The compressed image tiles are stored in the binary table in the same order that the first pixel in each tile appears in the FITS image; the tile containing the first pixel in the image appears in the first row of the table, and the tile containing the last pixel in the image appears in the last row of the binary table.

- ZNAMEn and ZVALn (optional keywords) These pairs of optional array keywords (where n is an integer index number starting with 1) supply the name and value, respectively, of any algorithm-specific parameters that are needed to compress or uncompress the image. The value of ZVALn may have any valid FITS datatype. The order of the compression parameters may be significant, and may be defined as part of the description of the specific decompression algorithm.

- ZMASKCMP (optional keyword) Used to record the name of the image compression algorithm that was used to compress the optional null pixel data mask. See the"Preserving undefined pixels with lossy compression" section for more details.

- The following 8 optional keywords are defined to store a verbatim copy of the the value and comment fields of the corresponding keywords in the original uncompressed FITS image. These keywords can be used to reconstruct an identical copy of the original FITS file when the image is uncompressed.

  - ZSIMPLE - preserves the original SIMPLE keyword
  - ZTENSION - preserves the original XTENSION keyword
  - ZEXTEND - preserves the original EXTEND keyword
  - ZBLOCKED - preserves the original BLOCKED keyword
  - ZPCOUNT - preserves the original PCOUNT keyword
  - ZGCOUNT - preserves the original GCOUNT keyword
  - ZHECKSUM - preserves the original CHECKSUM keyword
  - ZDATASUM - preserves the original DATASUM keyword

  The ZSIMPLE, ZEXTEND, and ZBLOCKED keywords may only be used if the original uncompressed image was contained in the primary array of the FITS file. The ZTENSION, ZPCOUNT, and ZGCOUNT keywords may only be used if the original uncompressed image was contained in in IMAGE extension.

- `Other Keywords` The FITS header of the compressed image may contain other optional keywords. If a FITS primary array or IMAGE extension is compressed using the convention described here, it is recommended that all the keywords in the header of the original image, except for the mandatory keywords mentioned above, be copied verbatim and in the same order into the header of the binary table extension that contains the compressed image. All these keywords will have the same meaning and interpretation as they did in the original image, even in cases where the keyword is not normally expected to occur in the header of a binary table extension (e.g., the `BSCALE` and `BZERO` keywords, or the World Coordinate System keywords such as `CTYPEn, CRPIXn` and `CRVALn`).

3 Columns

The following columns in the FITS binary table are defined by this convention. The order of the columns in the table is not significant. The column names (given by the `TTYPEn` keyword) are shown here in upper case letters, but the case is not significant.

- `COMPRESSED_DATA` (required column) Each row of this variable-length column contains the byte stream that was generated as a result of compressing the corresponding image tile. The datatype of the column (as given by the `TFORMn` keyword) will generally be either `'1PB'`, `'1PI'`, or `'1PJ'`, depending on whether the compression algorithm generates an output stream of 8-bit bytes, 16-bit integers, or 32-bit integers, respectively. If it is not possible to efficiently compress a particular image tile, then the `COMPRESSED_DATA` vector in the corresponding row will have a length of zero, and the uncompressed tile pixels will be written instead to the `UNCOMPRESSED_DATA` column described below.

- `UNCOMPRESSED_DATA` (optional column) This variable length column will contain the uncompressed pixels for any tiles that cannot be compressed. The datatype of this column will usually correspond to the datatype of the original image as shown in the following table:

Datatype	`BITPIX`	`TFORMn`
byte	8	'1PB'
short int	16	'1PI'
long int	32	'1PJ'
float	-32	'1PE'
double	-64	'1PD'

  If all the tiles in an image are compressed, then the `UNCOMPRESSED_DATA` column is not required.

- `ZSCALE and ZZERO` (optional columns) These columns give the linear scale factor and zero point offset which may be needed to transform the raw uncompressed values back to the original image pixel values (or at least a close approximation to the original values) using the following formula:

$$\text{image_pixel_value} = (\text{uncompressed_value} * \texttt{ZSCALE}) + \texttt{ZZERO}$$

  `ZSCALE` and `ZZERO` generally have double precision values and have default values of 1.0 and 0.0, respectively. If the same values of ZSCALE and ZZERO apply to every tile in the image, then they may be given as header keywords rather than as table columns.

ZSCALE and ZZERO are typically used to scale floating point images (with BITPIX = -32 or -64) into integers before compression, since most compression algorithms are not very efficient with floating point data. One particularly effective scaling algorithm is described in the next section.

These 2 parameters should not be confused with the reserved BSCALE and BZERO keywords which may be present in integer FITS images (which have BITPIX = 8, 16, or 32). Any such integer images should normally be compressed without any further scaling, and the BSCALE and BZERO keywords should be copied verbatim into the header of the binary table containing the compressed image.

- ZBLANK (optional column) In cases where floating point images are converted to integers before being compressed, this column gives the the integer value that is used to represent undefined pixels (if any) in the image. These pixels would have an IEEE NaN (Not a Number) value in the uncompressed floating point FITS image. If every tile uses the same null value, then ZBLANK may be given as a keyword instead of as a table column. If there are no undefined pixels in the image then ZBLANK is not required. If the uncompressed image has an integer datatype (ZBITPIX > 0) then the reserved BLANK keyword which already serves this purpose should be used instead of ZBLANK.

- NULL_PIXEL_MASK (optional column) In cases where the image contains undefined pixels and a lossy compression algorithm is used (and hence the pixel values are not exactly preserved) then this column is used to store a compressed image mask that records the location of any undefined pixels. See the "Preserving undefined pixels with lossy compression" section for more details.

- Other Columns Any number of other columns may be present in the table to supply other parameters that relate to each image tile.

4 Quantization of Floating Point Data

Images that have floating point data type pixels often do not compress very effectively due to the presence of 'noise' in the least significant bits of the pixel values. In order to achieve a higher degree of compression, one can effectively discard some of the 'noise' bits by linearly scaling the image into integer pixel values, so that

$$\text{floating_point_value} = (\text{integer_value} * \texttt{ZSCALE}) + \texttt{ZZERO}$$

The computational problem then becomes how to choose the most appropriate values for ZSCALE and ZZERO. The maximum amount of numerical precision will be preserved if the ZSCALE and ZZERO values are chosen so that the scaled pixel values span the full range of the integer datatype (e.g., from -32768 to +32767 for 16-bit integers), however this may also preserve an undesirable amount of non-significant noise, which can adversely affect the amount of compression that can be achieved.

A more effective scaling algorithm that throws away a specified amount of noise is described by White and Greenfield (in the Proceedings of the 1998 ADASS VIII conference). They first estimate the RMS noise in the image as 1.483 times the median of the absolute value of the differences of successive pixels. They then calculate the scaling factor so that the integer quantization level is $2^{-B}\sigma$, where B is the user-specified number of bits of accuracy in the quantization of the noise. By specifying the quantization level in terms of σ, this ensures that a given value of B will produce

comparable quality images regardless of the noise level. B is directly related to the compressed file size: increasing B by 1 will increase the file size by 1 bit/pixel. If this scaling algorithm has been applied to the image data before compression, then the following keywords should be present in the compressed image header:

```
ZNAMEn  = 'NOISEBIT'
ZVALn   = B
```

where B is the number of bits of accuracy (typically in the range 1 - 6).

Note that the tiled image compression convention described here does not require that floating point images be scaled to integers before compressing them, but if linear scaling is performed, then the ZSCALE and ZZERO columns in the FITS binary table should be used to record the 2 coefficients, as described in the previous section.

5 Preserving undefined pixels with lossy compression

Any undefined pixels in a FITS image are flagged with a special pixel value: the BLANK keyword specifies the value in integer data type FITS images, and an IEEE NaN (Not a Number) value is used in single or double precision floating point FITS images. Floating point images are often converted to scaled integers prior to compression (as described previously) in which case the undefined pixel value is then given by the ZBLANK keyword (or column).

The null pixel values in the image will be preserved if a lossless compression algorithm is used. If the image is compressed with a lossy algorithm (e.g., H-Compress with a scale factor greater than 1), then some other technique must be used to identify the null pixels in the image.

The recommended method of recording the null pixels when a lossy compression algorithm is used is to create an integer data mask with the same dimensions as the image tile. Set the null pixels to 1 and all the other pixels to 0, then compress the mask array using a lossless algorithm such as PLIO or GZIP. Store the compressed byte stream in a variable-length array column called 'NULL_PIXEL_MASK' in the row corresponding to that image tile. The ZMASKCMP keyword should be used to record the name of the algorithm used to compress the data mask (e.g., RICE_1). The data mask array pixels will be assumed to have the shortest integer datatype that is supported by the compression algorithm (i.e., usually 8-bit bytes).

When uncompressing the image tile, the software must check if the corresponding compressed data mask exists with a length greater than 0, and if so, then uncompress the mask and set the corresponding undefined pixels in the image array to the appropriate value (as given by the BLANK or ZBLANK keyword).

6 Currently Implemented Compression Algorithms

This section describes the 4 compression algorithms that are currently supported in the CFITSIO implementation of this tiled image compression convention (available from the HEASARC web site). This does not imply that other implementations of this convention must support these same algorithms, nor does it limit other implementations from supporting other compression algorithms.

6.1 Rice compression algorithm

The Rice algorithm (Rice, R. F., Yeh, P.-S., and Miller, W. H. 1993, in Proc. of the 9th AIAA Computing in Aerospace Conf., AIAA-93-4541-CP, American Institute of Aeronautics and Astronautics)

is simple and very fast, compressing or decompressing 10^7 pixels/sec on modern workstations. It requires only enough memory to hold a single block of 16 or 32 pixels at a time. It codes the pixels in small blocks and so is able to adapt very quickly to changes in the input image statistics (e.g., Rice has no problem handling cosmic rays, bright stars, saturated pixels, etc.).

The block size that is used should be recorded in the compressed image header with

```
ZNAMEn  = 'BLOCKSIZE'
ZVALn   = 16 or 32
```

If these keywords are absent, then a default blocksize of 32 should be assumed.

6.2 GZIP compression algorithm

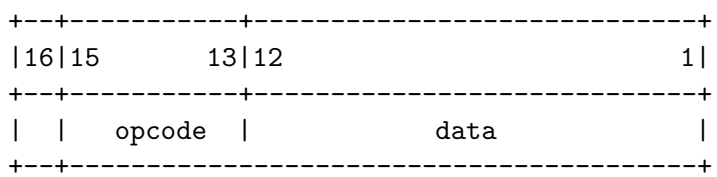
Gzip is the compression algorithm used in the widely distributed GNU free software utility of the same name. It was created by Jean-loup Gailly and Mark Adler. Version 0.1 was first publicly released on October 31, 1992. Version 1.0 followed in February 1993. It is based on the DEFLATE algorithm, which is a combination of LZ77 and Huffman coding. DEFLATE was intended as a replacement for LZW and other patent-encumbered data compression algorithms which, at the time, limited the usability of compress and other popular archivers. Further information about this compression technique is readily available on the Internet.

The gzip algorithm has no associated parameters that need to be specified with the `ZNAMEn and ZVALn` keywords.

6.3 IRAF PLIO compression algorithm

The IRAF PLIO (pixel list) algorithm was developed to store integer-valued image masks in a compressed form. Typical uses of image masks are to segment images into regions, or to mark bad pixels. Such masks often have large regions of constant value hence are highly compressible. The compression algorithm used is based on run-length encoding, with the ability to dynamically follow level changes in the image, allowing a 16-bit encoding to be used regardless of the image depth. The worst case performance occurs when successive pixels have different values. Even in this case the encoding will only require one word (16 bits) per mask pixel, provided either the delta intensity change between pixels is usually less than 12 bits, or the mask represents a zero floored step function of constant height. The worst case cannot exceed npix*2 words provided the mask depth is 24 bits or less.

A good compromise between storage efficiency and efficiency of runtime access, while keeping things simple, is achieved if we maintain the compressed line lists as variable length arrays of type short integer (16 bits per list element), regardless of the mask depth. A line list consists of a series of simple instructions which are executed in sequence to reconstruct a line of the mask. Each 16 bit instruction consists of the sign bit (not used at present), a three bit opcode, and twelve bits of data, i.e.:



The significance of the data depends upon the instruction. The instructions currently implemented are summarized in the table below.

Instruction	Opcode	Description
ZN	00	Output N zeros
HN	04	Output N high values
PN	05	Output N-1 zeros plus one high value
SH	01	Set high value, absolute
IH,DH	02,03	Increment or decrement high value
IS,DS	06,07	Like IH-DH, plus output one high value

In order to reconstruct a mask line, the application executing these instructions is required to keep track of two values, the current high value and the current position in the output line. The detailed operation of each instruction is as follows:

ZN Zero the next N (=data) output pixels.

HN Set the next N output pixels to the current high value.

PN Zero the next N-1 output pixels, and set pixel N to the current high value.

SH Set the high value (absolute rather than incremental), taking the high 15 bits from the next word in the instruction stream, and the low 12 bits from the current data value.

IH,DH Increment (IH) or decrement (DH) the current high value by the data value. The current position is not affected.

IS,DS Increment (IS) or decrement (DS) the current high value by the data value, and step, i.e., output one high value.

The high value is assumed to be set to 1 at the beginning of a line, hence the IH,DH and IS,DS instructions are not normally needed for boolean masks. If the length of a line segment of constant value or the difference between two successive high values exceeds 4096 (12 bits), then multiple instructions are required to describe the segment or intensity change.

6.4 H-Compress algorithm

HCOMPRESS is an the image compression package written by Richard L. White for use at the Space Telescope Science Institute (rlw@stsci.edu). Hcompress was used to compress the STScI Digitized Sky Survey and has also been used to compress the preview images in the Hubble Data Archive. Briefly, the method used is:

1. a wavelet transform called the H-transform (a Haar transform generalized to two dimensions), followed by

2. quantization that discards noise in the image while retaining the signal on all scales, followed by

3. quadtree coding of the quantized coefficients.

The technique gives very good compression for astronomical images and is relatively fast. The calculations are carried out using integer arithmetic and are entirely reversible. Consequently, the program can be used for either lossy or lossless compression, with no special approach needed for the lossless case (e.g. there is no need for a file of residuals.)

There are 2 user-defined parameters associated with the H-Compress algorithm: an integer scale factor that determines the amount of compression, and a Boolean parameter the specifies whether the image should be smoothed during the decompression operation, to reduce residual artifacts in the image.

- **Scale Factor.** The integer scale parameter determines the amount of compression. Scale = 0 or 1 leads to lossless compression, i.e. the decompressed image has exactly the same pixel values as the original image. If the scale factor is greater than 1 then the compression is lossy: the decompressed image will not be exactly the same as the original. For astronomical images, lossless compression is generally rather ineffective because the images have a good deal of noise, which is inherently incompressible. However, if some of this noise is discarded then the images compress very well. The scale factor determines how much of the noise is discarded. Setting scale to 2 times sigma, the RMS noise in the image, usually results in compression by about a factor of 10 (i.e. the compressed image requires about 1.5 bits/pixel), while producing a decompressed image that is nearly indistinguishable from the original. In fact, the RMS difference between the decompressed image and the original image will be only about 1/2 sigma. Experiments indicate that this level of loss has no noticeable effect on either the visual appearance of the image or on quantitative analysis of the image (e.g. measurements of positions and brightnesses of stars are not adversely affected.)

  Using a larger value for scale results in higher compression at the cost of larger differences between the compressed and original images. A rough rule of thumb is that if scale equals N sigma, then the image will compress to about 3/N bits/pixel, and the RMS difference between the original and the compressed image will be about N/4 sigma. This crude relationship is inaccurate both for very high compression ratios and for lossless compression, but it does at least give an indication of what to expect of the compressed images.

  For images in which the noise varies from pixel to pixel (e.g. CCD images, where the noise is larger for brighter pixels), the appropriate value for scale is determined by the RMS noise level in the sky regions of the image. For images that are essentially noiseless, any lossy compression is noticeable under sufficiently close inspection of the image, but some loss is nonetheless acceptable for typical applications. Note that the quantization scheme used in hcompress is not designed to give images that appear as much like the original as possible to the human eye, but rather is designed to produce images that are as similar as possi- ble to the original under quantitative analysis. Thus, the emphasis is on discarding noise without affecting the signal rather than on discarding components of the image that are not very noticeable to the eye (as may be done, for example, by JPEG compression.) The resulting compression scheme is not ideal for typical terrestrial images (though it is still a reasonably good method for those images), but is believed to be close to optimal for astronomical images.

  It is not necessary to know what scale factor was used when compressing the image in order to uncompress it, but it is still useful to record the value that was used. It is recommended that the `ZNAMEn` and `ZVALn)` pair of keywords be used for this purpose, with

  ```
  ZNAMEn = 'SCALE'
  ZVALn  =  I
  ```

  where I is the integer scale value.

- **Smoothing Flag.** At high compressions factors the decompressed image begins to appear blocky because of the way information is discarded. This blockiness ness is greatly reduced,

producing more pleasing images, if the image is smoothed slightly during decompression. When done properly, the smoothing will not affect any quantitative photometric or astrometric measurements derived from the compressed image. Of course, the smoothing should never be applied when the image has been losslessly compressed with a scale factor (defined above) of 0 or 1.

The smoothing option only needs to be specified when uncompressing the image, however, in many cases, this can best be determined by the person or project that creates the compressed image files. Thus it is recommended that the smoothing flag be specified in the compressed image header with the `ZNAMEn` and `ZVALn` keywords with

```
ZNAMEn = 'SMOOTH'
ZVALn  =  0 or 1
```

A value of 0 means no smoothing, and any other value means smoothing is recommended. This should be regarded as only a recommendation which the image decompression program may override.

A paper describing HCompress was published in the Proceedings of the NASA Space and Earth Science Data Compression Workshop, ed. James C. Tilton, Snowbird, Utah, March 1992. This paper is reproduced in the following appendix of this document.

A Appendix

High-Performance Compression of Astronomical Images

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Summary

Astronomical images have some rather unusual characteristics that make many existing image compression techniques either ineffective or inapplicable. A typical image consists of a nearly flat background sprinkled with point sources and occasional extended sources. The images are often noisy, so that lossless compression does not work very well; furthermore, the images are usually subjected to stringent quantitative analysis, so any lossy compression method must be proven not to discard useful information, but must instead discard only the noise. Finally, the images can be extremely large. For example, the Space Telescope Science Institute has digitized photographic plates covering the entire sky, generating 1500 images each having 14000×14000 16-bit pixels. Several astronomical groups are now constructing cameras with mosaics of large CCDs (each 2048×2048 or larger); these instruments will be used in projects that generate data at a rate exceeding 100 MBytes every 5 minutes for many years.

An effective technique for image compression may be based on the H-transform/ (Fritze et al. 1977). The method that we have developed can be used for either lossless or lossy compression. The digitized sky survey images can be compressed by at least a factor of 10 with no noticeable losses in the astrometric and photometric properties of the compressed images. The method has been designed to be computationally efficient: compression or decompression of a 512×512 image requires only 4 seconds on a Sun SPARCstation 1. The algorithm uses only integer arithmetic, so it is completely reversible in its lossless mode, and it could easily be implemented in hardware for space applications.

1. Introduction

Astronomical images consist largely of empty sky. Compression of such images can reduce the volume of data that it is necessary to store (an important consideration for large scale digital sky surveys) and can shorten the time required to transmit images (useful for remote observing or remote access to data archives.)

Data compression methods can be classified as either "lossless" (meaning that the original data can be reconstructed exactly from the compressed data) or "lossy" (meaning that the uncompressed image is not exactly the same as the original.) Astronomers often insist that they can accept only lossless compression, in part because of conservatism, and in part because the familiar lossy compression methods sacrifice some information that is needed for accurate analysis of image data. However, since all astronomical images contain noise, which is inherently incompressible, lossy compression methods produce much better compression results.

A simple example may make this clear. One of the simplest data compression techniques is run-length coding, in which runs of consecutive pixels having the same value are compressed by storing the pixel value and the repetition factor. This method is used in the standard compression scheme for facsimile transmissions. Unfortunately, it is quite ineffective for lossless compression of

astronomical images because even though the sky is *nearly* constant, the noise in the sky ensures that only very short runs of equal pixels occur. The obvious way to make run-length coding more effective is to force the sky to be exactly constant by setting all pixels below a threshold (chosen to be just above the sky) to the mean sky value. However, then one has lost any information about objects close to the detection limit. One has also lost information about local variations in the sky brightness, which severely limits the accuracy of photometry and astrometry on faint objects. Worse, there may be extended, low surface brightness objects that are not detectable in a single pixel but that are easily detected when the image is smoothed over a number of pixels; such faint structures are irretrievably lost when the image is thresholded to improve compression.

2. The H-transform

Fritze et al. (1977; see also Richter 1978 and Capaccioli et al. 1988) have developed a much better compression method for astronomical images based on what they call the *H-transform* of the image. A similar transform called the S-transform has also been used for image compression (Blume & Fand 1989). The H-transform is a two-dimensional generalization of the Haar transform (Haar 1910). The H-transform/ is calculated for an image of size $2^N \times 2^N$ as follows:

- Divide the image up into blocks of 2×2 pixels. Call the 4 pixels in a block a_{00}, a_{10}, a_{01}, and a_{11}.
- For each block compute 4 coefficients:

  $h_0 = (a_{11} + a_{10} + a_{01} + a_{00})/2$
  $h_x = (a_{11} + a_{10} - a_{01} - a_{00})/2$
  $h_y = (a_{11} - a_{10} + a_{01} - a_{00})/2$
  $h_c = (a_{11} - a_{10} - a_{01} + a_{00})/2$

- Construct a $2^{N-1} \times 2^{N-1}$ image from the h_0 values for each 2×2 block. Divide that image up into 2×2 blocks and repeat the above calculation. Repeat this process N times, reducing the image in size by a factor of 2 at each step, until only one h_0 value remains.

This calculation can be easily inverted to recover the original image from its transform. The transform is exactly reversible using integer arithmetic if one does not divide by 2 for the first set of coefficients. It is straightforward to extend the definition of the transform so that it can be computed for non-square images that do not have sides that are powers of 2. The H-transform can be performed in place in memory and is very fast to compute, requiring about $16M^2/3$ (integer) additions for a $M \times M$ image.

The H-transform is a simple 2-dimensional wavelet transform. It has several advantages over some other wavelet transforms that have been applied to image compression (e.g., Daubechies 1988). First, the transform can be performed entirely with integer arithmetic, making it exactly reversible. Consequently it can be used for either lossless or lossy compression (as indicated below) and one does not need a special technique for the case of lossless compression (as was required, e.g., for the JPEG compression standard.)

A second major advantage is that the H-transform is a natively 2-dimensional wavelet transform. The standard 1-dimensional wavelet transforms are extended to two dimensions by transforming the image first along the rows, then along the columns. Unfortunately, this generates many wavelet coefficients that are high frequency (hence localized) in the x-direction but low frequency (hence global) in the y-direction. Such coefficients are counter to the philosophy of the wavelet transform: high-frequency basis functions should be confined to a relatively small area of the image.

Discarding these mixed-scale terms, which may be negligible compared to the noise, generates very objectionable artifacts around point sources and edges in the image. The H-transform, on the other hand, is a fully 2-dimensional wavelet transform, with all high frequency terms being completely localized. It is consequently more suitable for image compression and produces fewer artifacts.

A possible disadvantage of the H-transform is that other wavelet transforms take better advantage of the continuity of pixel values within images, so that they can produce higher compressions for very smooth images. However, for astromical images (which are mostly flat sky sprinkled with point sources) the smoothness built into higher-order transforms can actually reduce the effectiveness of compression, because one must keep more coefficients to describe each point source.

3. Compression Using the H-transform

If the image is nearly noiseless, the H-transform is somewhat easier to compress than the original image because the differences of adjacent pixels (as computed in the H-transform) tend to be smaller than the original pixel values for smooth images. Consequently fewer bits are required to store the values of the H-transform coefficients than are required for the original image. For very smooth images the pixel values may be constant over large regions, leading to transform coefficients that are zero over large areas.

Noisy images still do not compress well when transformed, though. Suppose there is noise σ in each pixel of the original image. Then from simple propagation of errors, the noise in each of the H-transform coefficients is also σ. To compress noisy images, divide each coefficient by $S\sigma$, where $S \sim 1$ is chosen according to how much loss is acceptable. This reduces the noise in the transform to $0.5/S$, so that large portions of the transform are zero (or nearly zero) and the transform is highly compressible.

Why is this better than simply thresholding the original image? As discussed above, if we simply divide the image by σ then we lose all information on objects that are within 1σ of sky in a *single* pixel, but that are detectable by averaging a *block* of pixels. On the other hand, in dividing the H-transform by σ, we preserve the information on any object that is detectable by summing a block of pixels! The quantized H-transform preserves the mean of the image for every block of pixels having a mean significantly different than that of neighboring blocks of pixels.

As an example, Figure 1 shows a 128×128 section (3.6×3.6 arcmin) from a digitized version of the Palomar Observatory–National Geographic Society Sky Survey plate containing the Coma cluster of galaxies. Figures 2, 3, and 4 show the resulting image for $S \simeq 0.5$, 1, and 2. These images are compressed by factors of 10, 20, and 60 using the coding scheme described below. In all cases a logarithmic gray scale is used to show the maximum detail in the image near the sky background level; the noise is clearly visible in Figure 1. The image compressed by a factor of 10 is hardly distinguishable from the original. In quantizing the H-transform we have adaptively filtered the original image by discarding information on some scales and keeping information on other scales. This adaptive filtering is most apparent for high compression factors (Fig. 4), where the sky has been smoothed over large areas while the images of stars have hardly been affected.

The adaptive filtering is, in itself, of considerable interest as an analytical tool for images (Capaccioli et al. 1988). For example, one can use the adaptive smoothing of the H-transform to smooth the sky without affecting objects detected above the (locally determined) sky; then an accurate sky value can be determined by reference to any nearby pixel.

The blockiness that is visible in Figure 4 is the result of difference coefficients being set to zero over large areas, so that blocks of pixels are replaced by their averages. It is possible to eliminate the blocks by an appropriate filtering of the image. A simple but effective filter can be derived by simply adjusting the H-transform coefficients as the transform is inverted to produce a smooth image; as long as changes in the coefficients are limited to $\pm S\sigma/2$, the resulting image will still be

consistent with the thresholded H-transform.

4. Efficient Coding

The quantized H-transform has a rather peculiar structure. Not only are large areas of the transform image zero, but the non-zero values are strongly concentrated in the lower-order coefficients. The best approach we have found to code the coefficient values efficiently is quadtree coding of each bitplane of the transform array. Quadtree coding has been used for many purposes (see Samet 1984 for a review); the particular form we are using was suggested by Huang and Bijaoui (1991) for image compression.

- Divide the bitplane up into 4 quadrants. For each quadrant code a '1' if there are any 1-bits in the quadrant, else code a '0'.
- Subdivide each quadrant that is not all zero into 4 more pieces and code them similarly. Continue until one is down to the level of individual pixels.

This coding (which Huang and Bijauoi call "hierarchic 4-bit one" coding) is obviously very well suited to the H-transform image because successively lower orders of the H-transform coefficients are located in successively divided quadrants of the image.

We follow the quadtree coding with a fixed Huffman coding that uses 3 bits for quadtree values that are common (e.g., 0001, 0010, 0100, and 1000) and uses 4 or 5 bits for less common values. This reduces the final compressed file size by about 10% at little computational cost. Slightly better compression can be achieved by following quadtree coding with arithmetic coding (Witten, Bell, and Cleary 1987), but the CPU costs of arithmetic coding are not, in our view, justified for 3–4% better compression. We have also tried using arithmetic coding directly on the H-transform, with various contexts of neighboring pixels, but find it to be both computationally inefficient and not significantly better than quadtree coding.

For completely random bitplanes, quadtree coding can actually use more storage than simply writing the bitplane directly; in that case we just dump the bitplane with no coding.

Note that by coding the transform one bitplane at a time, the compressed data can be viewed as an incremental description of the image. One can initially transmit a crude representation of the image using only the small amount of data that is required for the sparsely populated, most significant bit planes. Then the lower bit planes can be added one by one until the desired accuracy is required. This could be useful, for example, if the data is to be retrieved from a remote database — one could examine the crude version of the image (retrieved very quickly) and abort the transmission of the rest of the data if the image is judged to be uninteresting.

5. Astrometric and Photometric Properties of Compressed Images

Astronomical images are not simply subjected to visual examination, but are also subjected to careful quantitative analysis. For example, for the image in Figure 1 one would typically like to do astrometric (positional) measurements of objects to an accuracy much better than 1 pixel, photometric (brightness) measurements of objects to an accuracy limited only by the detector response and the noise, and accurate measurements of the surface brightness of extended sources.

We have done some experiments to study the degradation of astrometry and photometry on the compressed images compared to the original images (White, Postman, and Lattanzi 1991). Even the most highly compressed images have very good photometric properties for both point sources and extended sources; indeed, photometry of extended objects can be improved by the adaptive filtering of the H-transform (Cappacioli et al. 1988). Astrometry is hardly affected by the compression for modest compression factors (up to about a factor of 20 for our digitized photographic plates), but does begin to degrade for the most highly compressed images.

These results are based on tests carried out with tools optimized for the original images; it is likely the best results will be obtained for highly compressed images only with analysis tools specifically adapted to the peculiar noise characteristics of the compressed images.

6. Conclusions

In order to construct the Guide Star Catalog for use in pointing the Hubble Space Telescope, the Space Telescope Science Institute scanned and digitized wide-field photographic plates covering the entire sky. The digitized plates are of great utility, but to date it has been impossible to distribute the scans because of the massive volume of data involved (a total of about 600 Gbytes). Using the compression techniques described in this paper, we plan to distribute our digital sky survey on CD-ROMs; about 100 CD-ROMs will be required if the survey is compressed by a factor of 10.

The algorithm described in this paper has been shown to be capable of producing highly compressed images that are very faithful to the original. Algorithms designed to work on the original images can give comparable results on object detection, astrometry, and photometry when applied to the images compressed by a factor of 10 or possibly more. Further experiments will determine more precisely just what errors are introduced in the compressed data; it is possible that certain kinds of analysis will give more accurate results on the compressed data than on the original because of the adaptive filtering of the H-transform (Capaccioli et al. 1988).

This compression algorithm can be applied to any image, not just to digitized photographic plates. Experiments on CCD images indicate that lossless compression factors of 3–30 can be achieved depending on the CCD characteristics (e.g., the readout noise). A slightly modified algorithm customized to the noise characteristics of the CCD will do better. This application will be explored in detail in the future.

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